Price-setting Mixed Duopoly, Partial Privatization and Subsidization: Substitute and Complementary Goods

Kazuhiro Ohnishi*

Institute for Economic Sciences, Japan. E-mail: ohnishi@e.people.or.jp

Abstract: This paper examines partial privatization in a price-setting mixed duopoly model to reassess the welfare effect of production subsidies. The paper considers both substitute and complementary goods. The paper demonstrates that the result of price competition with complementary goods is essentially the same as that of price competition with substitute goods. The paper also demonstrates that the optimal subsidy, output and economic welfare are higher in price competition with complementary goods than in price competition with substitute goods.

Keywords: Partial privatization; Price competition; Subsidization; Complementary goods

JEL classification: C72; D21; L32

1. Introduction

Recently, many researchers have done a lot of work on privatization of public firms (e.g., Gronberg and Hwang, 1992; Anderson et al., 1997; Bosi et al., 2005; Chang, 2005; Chao and Yu, 2006; Han and Ogawa, 2008; Capuano and De Feo, 2010; Ohnishi, 2012; Bárcena-Ruiz and Garzón, 2017). White (1996) investigates how production subsidies influence the privatization decision in a quantity-setting mixed oligopoly market and presents the following three main results. First, if production subsidies are utilized before and after privatization, the level of economic welfare is not changed. Second, if production subsidies are utilized only before privatization, there is a reduction in economic welfare. Third, the production subsidy contributes to overall efficiency in a mixed oligopoly market because of cost distribution effects. Poyago-Theotoky (2001) and Myles (2002) show that the optimal production subsidy is identical regardless of whether (i) a public firm moves simultaneously with n private firms, (ii) it is a Stackelberg leader, or (iii) all firms are profit-maximisers. These are called "irrelevance results".

The analysis by Fershtman (1990) examined a mixed duopoly model in which the government owned a partial share of a firm that was the Cournot competitor of a private firm. Since then, many researchers have contributed

to the theoretical analysis of partial privatization of state-owned public firms (e.g., Matsumura, 1998; Lu and Poddar, 2007; Saha and Sensarma, 2008; Artz et al., 2009; Wang et al., 2009; Heywood and Ye, 2010; Ohnishi, 2010b, 2016; Wang and Lee, 2010; Chen, 2017; Heywood et al., 2017; Fridman, 2018). Tomaru (2006) studies partial privatization in quantity-setting mixed oligopoly competition with subsidies and shows that the optimal subsidy and economic welfare are identical irrespective of the level of privatization of a public firm. In addition, Scrimitore (2014) studies partial privatization in quantity and price competition between a public firm and a private firm under optimal subsidies and highlights the equivalence between a quantity (resp. price) game with public leadership or simultaneous moves and a price (resp. quantity) game with private leadership.

In the present paper, we examine a price-setting mixed market model with substitute and complementary goods to reassess the subsidy effect of partial privatization. The case of complementary goods has lots of examples such as black pens and red pens. We demonstrate that the result of price competition with complementary goods is essentially the same as that of price competition with substitute goods. We find that we can obtain an irrelevance result even though complementary goods are considered.

The rest of this paper proceeds as follows. In Section 2, we describe the basic setting. Section 3 presents the result of price-setting competition with substitute goods. Section 4 examines the analysis for price competition with complementary goods. Section 5 compares the result obtained in Section 3 with that in Section 4. Finally, Section 6 concludes the paper.

2. Basic setting

Consider an industry composed of a private firm (firm 1) and a partially privatized firm (firm 0) that is jointly owned by both the public and private sectors. Both firms produce imperfectly substitutable goods. Throughout this paper, subscripts 0 and 1 represent firm 0 and firm 1, respectively. In addition, when i and j are used to represent firms in an expression, they should be understood to refer to 0 and 1 with $i \neq j$. We do not consider the possibility of entry or exit. The basic setting is taken from Bárcena-Ruiz and Sedano (2011). Firm i's demand function is given by

$$q_i = \frac{a(1-b) - p_i + bp_j}{1 - b^2} \tag{1}$$

where $a \in (0, \infty)$ is a constant, $b \in (-1, 1)$ is a measure of the degree of complementarity/substitutability among products, and $p_i \in (0, \infty)$ is firm i's price.

Firm i's profit is given by

$$\pi_i = (p_i - c + s) q_i \tag{2}$$

where $c \in (0, a)$ denotes the total cost for each unit of output and $s \in (0, \infty)$ is the subsidy for each unit of output. Firm 1 aims to maximize (2).

Economic welfare is given by

$$W = CS + \pi_0 + \pi_1 - s(q_0 + q_1) \tag{3}$$

where $CS = [p_0^2 - 2bp_0 p_1 + p_1^2 + 2a(1-b)(a - p_0 - p_1)] / 2(1 - b^2)$ represents consumer surplus.

Firm 0's objective function is given by

$$U_0 = \lambda W + (1 - \lambda)\pi_0$$
$$= \lambda \left[\frac{p_0^2 - 2bp_0 p_1 + p_1^2 + 2a(1-b)(a - p_0 - p_1)}{2(1 - b^2)} + (p_0 - c)q_0 + (p_1 - c)q_1 \right] + (1 - \lambda)(p_0 - c + s)q_0$$

$$\tag{4}$$

where $\lambda \in [0,1]$ represents the level of privatization. That is, if $\lambda = 0$ firm 0 is purely private, whereas if $\lambda = 1$ it is purely public.

The game model has two stages. In the first stage, the government sets the production subsidy to maximize economic welfare for a given λ. In the second stage, both firms simultaneously and independently choose their prices. In this paper, we solve for the subgame perfect equilibrium through backward induction.

3. Price Competition with Substitute Goods

In this section, we examine a mixed duopoly game in which both firms produce substitute goods. Therefore, we assume $b = 0.5$. Consumer surplus is rewritten as follows:

$$CS^S = \frac{2}{3} \left[p_0^2 - p_0 p_1 + p_1^2 + a(a - p_0 - p_1) \right]$$

where the superscript "S" denotes price competition with substitute goods. Therefore, firm 0's objective function is as follows:

$$U_0^S = \lambda \left\{ \frac{2}{3} \left[p_0^2 - p_0 p_1 + p_1^2 + a(a - p_0 - p_1) \right] + (p_0 - c)q_0 + (p_1 - c)q_1 \right\} + (1 - \lambda)(p_0 - c + s)q_0$$

$$\tag{5}$$

As usual, the game is solved by backward induction. Starting from the second stage, we obtain the two firms' reaction functions in prices :

$$R_0^S(p_1) = \frac{a + 2c - 2s - a\lambda - c\lambda + 2s\lambda + p_1}{4 - 2\lambda} \tag{6}$$

$$R_1^s(p_0) = \frac{a + 2c - 2s + p_0}{4} \tag{7}$$

Furthermore, from (6) and (7), we derive the second-stage equilibrium prices in terms of s:

$$p_0^s(s, \lambda) = \frac{5a + 10c - 10s - 4a\lambda - 4c\lambda + 8s\lambda}{15 - 8\lambda} \tag{8}$$

$$p_1^s(s, \lambda) = \frac{5a + 10c - 10s - 3a\lambda - 5c\lambda + 6s\lambda}{15 - 8\lambda} \tag{9}$$

When setting $s = 0$ in (8) and (9), we obtain $p_0^s(0, \lambda) \le p_1^s(0, \lambda)$ with equality if firm 0 is completely privatized. As the level of λ rises from 0 to 1, firm 0 becomes more interested in consumer surplus. Firm 0 has an incentive to charge a lower price than firm 1 and to sell more than firm 1 so as to raise consumer surplus. Therefore, without the production subsidy, firm 0's price is lower than firm 1's price while firm 0's output exceeds firm 1's output.

We now consider the first stage of the game. In the first stage, taking into account how firms will react to the subsidy, the government sets the subsidy so as to maximize (3). We obtain the welfare-maximizing subsidy as follows:

$$s^s = \frac{a - c}{2} \tag{10}$$

We obtain the following subgame perfect equilibrium values:

$$p_0^s(s^s, \lambda) = p_1^s(s^s, \lambda) = c \tag{11}$$

$$q_0^s(s^s, \lambda) = q_1^s(s^s, \lambda) = \frac{2(a - c)}{3} \tag{12}$$

$$\pi_0^s(s^s, \lambda) = \pi_1^s(s^s, \lambda) = \frac{(a - c)^2}{3} \tag{13}$$

$$W^s(s^s, \lambda) = \frac{2(a - c)^2}{3} \tag{14}$$

Note that the optimal subsidy achieves the first-best outcome in which price equals marginal cost. Also note that the equilibrium values do not depend on λ. Now we can state the following proposition.

Proposition 1: Suppose that both firms produce substitute goods. Then the optimal subsidy, economic welfare, and firms' profits are identical regardless of the privatization level of firm 0.

Proposition 1 indicates that this result is the same as that of price competition with substitute goods obtained by Scrimitore (2014).

4. Price Competition with Complementary Goods

In this section, we discuss a price-setting mixed duopoly game with complementary goods. Hence, we assume $b = -0.5$. Consumer surplus is rewritten as follows:

$$CS^C = \frac{2}{3}\left[p_0^2 + p_0 p_1 + p_1^2 + 3a\left(a - p_0 - p_1\right)\right]$$

where the superscript "C" denotes price competition with complementary goods. Therefore, firm 0's objective function is

$$U_0^C = \lambda\left\{\frac{2}{3}\left[p_0^2 + p_0 p_1 + p_1^2 + 3a\left(a - p_0 - p_1\right)\right] + \left(p_0 - c\right)q_0 + \left(p_1 - c\right)q_1\right\} + \left(1 - \lambda\right)\left(p_0 - c + s\right)q_0$$

(15)

We obtain the two firms' reaction functions with complementary goods :

$$R_0^C(p_1) = \frac{3a + 2c - 2s - 3a\lambda + c\lambda + 2s\lambda - p_1}{4 - 2\lambda}$$

(16)

$$R_1^C(p_0) = \frac{3a + 2c - 2s - p_0}{4}$$

(17)

Furthermore, from (16) and (17), we derive the second-stage equilibrium prices in terms of s :

$$p_0^C(s, \lambda) = \frac{9a + 6c - 6s - 12a\lambda + 4c\lambda + 8s\lambda}{15 - 8\lambda}$$

(18)

$$p_1^C(s, \lambda) = \frac{9a + 6c - 6s - 3a\lambda - 5c\lambda + 2s\lambda}{15 - 8\lambda}$$

(19)

When setting $s = 0$ in (18) and (19), we have $p_0^C(0, \lambda) \le p_1^C(0, \lambda)$. This is the same as the case of substitute goods.

In stage one, the government sets the subsidy to maximize (3). Therefore, we obtain the following welfare-maximizing subsidy:

$$s^C = \frac{3(a - c)}{2}$$

(20)

Note that S^C is three times higher than S^s. We have the following subgame perfect equilibrium values:

$$p_0^C(s^C, \lambda) = p_1^C(s^C, \lambda) = c \tag{21}$$

$$q_0^C(s^C, \lambda) = q_1^C(s^C, \lambda) = 2(a-c) \tag{22}$$

$$\pi_0^C(s^C, \lambda) = \pi_1^C(s^C, \lambda) = 3(a-c)^2 \tag{23}$$

$$W^C(s^C, \lambda) = 2(a-c)^2 \tag{24}$$

We present the following proposition.

Proposition 2: Suppose that both firms produce complementary goods. Then the optimal subsidy, economic welfare, and firms' profits are identical regardless of the privatization level of firm 0.

Proposition 2 means that the result of price competition with complementary goods is essentially the same as that of price competition with substitute goods.

5. Comparisons

In this section, we compare the equilibrium values for substitute goods with those for complementary. These comparisons can be depicted as follows:

$$s^C - s^S = a - c$$

$$p_i^C - p_i^S = 0$$

$$q_i^C - q_i^S = \frac{4(a-c)}{3}$$

$$W^C - W^S = \frac{4(a-c)^2}{3}$$

Although p_i^C is equal to p_i^S, s^C is higher than s^s. This leads to higher q_i^C and W^C. This result can be summarized in the following proposition.

Proposition 3: The optimal subsidy, output and economic welfare are higher in price competition with complementary goods than in price competition with substitute goods, while prices are identical for these two games.

6. Conclusion

We have examined partial privatization in a price-setting mixed duopoly game to reassess the welfare effect of production subsidies. We have considered both substitute and complementary goods. We have shown that the result of price competition with complementary goods is essentially the same as that of price competition with substitute goods. We have also shown that the

optimal subsidy, output and economic welfare are higher in price competition with complementary goods than in price competition with substitute goods.

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